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Zadanie 1.

Oblicz pole odcinka paraboli $y = x^2$ o cięciwie AB, jeśli

a) $A = (0; 0); B = (2; 4)$
b) $A = (1; 1); B = (3; 9)$
c) $A = (\pi;\ \pi^2); B = (4; 16)$
d) $A = (\sqrt{2}; 2); B = (\sqrt{5}; 5)$

Rozwiązanie

a)

$$a = \frac{4}{2} = 2$$

$$y' = 2x$$

$$2x = 2$$

$$x = 1$$

$$y = 1$$

$$C = (1; 1)$$

$$P_{ABC} = \frac{1}{2}\left\| \begin{matrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 4 & 1 \end{matrix} \right\| = \frac{1}{2}|2 - 4| = \frac{1}{2} \cdot |-2| = 1$$

$$P = \frac{4}{3} \cdot P_{ABC} = \frac{4}{3} \cdot 1 = \frac{4}{3}$$

b)

$$a = \frac{8}{2} = 4$$

$$y' = 2x$$

$$2x = 4$$

$$x = 2$$

$$y = 4$$

$$C = (2; 4)$$

$$P_{ABC} = \frac{1}{2}\left\| \begin{matrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 9 & 4 \end{matrix} \right\| = \frac{1}{2}|12 + 2 + 9 - 3 - 18 - 4| = \frac{1}{2} \cdot |-2| = 1$$

$$P = \frac{4}{3} \cdot P_{ABC} = \frac{4}{3} \cdot 1 = \frac{4}{3}$$

c)

$$a = \frac{16 - \pi^2}{4 - \pi} = \frac{(4 - \pi)(4 + \pi)}{4 - \pi} = 4 + \pi$$

$$y' = 2x$$

$$2x = 4 + \pi$$

$$x = 2 + \frac{\pi}{2}$$

$$y = \left(2 + \frac{\pi}{2}\right)^2 = 4 + 2\pi + \frac{\pi^2}{4}$$

$$C = \left(2 + \frac{\pi}{2}; 4 + 2\pi + \frac{\pi^2}{4}\right)$$

$$P_{ABC} = \frac{1}{2}\left\|\begin{matrix} 1 & 1 & 1 \\ \pi & 4 & 2 + \frac{\pi}{2} \\ \pi^2 & 16 & 4 + 2\pi + \frac{\pi^2}{4} \end{matrix}\right\| =$$

$$= \frac{1}{2}\left|16 + 8\pi + \pi^2 + 2\pi^2 + \frac{\pi^3}{2} + 16\pi - 4\pi^2 - 32 - 8\pi - 4\pi - 2\pi^2 - \frac{\pi^3}{4}\right| =$$

$$= \frac{1}{2}\left|\frac{\pi^3}{4} - 3\pi^2 + 12\pi - 16\right| = \frac{1}{2}\left(-\frac{\pi^3}{4} + 3\pi^2 - 12\pi + 16\right) =$$

$$= -\frac{\pi^3}{8} + \frac{3\pi^2}{2} - 6\pi + 8$$

$$P = \frac{4}{3}P_{ABC} = \frac{4}{3}\left(-\frac{\pi^3}{8} + \frac{3\pi^2}{2} - 6\pi + 8\right) = -\frac{\pi^3}{6} + 2\pi^2 - 8\pi + \frac{32}{3}$$

d)

$$a = \frac{16 - \pi^2}{\sqrt{5} - \sqrt{2}} = \frac{3\sqrt{5} + 3\sqrt{2}}{3} = \sqrt{5} + \sqrt{2}$$

$$y' = 2x$$

$$2x = \sqrt{5} + \sqrt{2}$$

$$x = \frac{\sqrt{5} + \sqrt{2}}{2}$$

$$y = \left(\frac{\sqrt{5} + \sqrt{2}}{2}\right)^2 = \frac{5 + 2\sqrt{10} + 2}{4} = \frac{7 + 2\sqrt{10}}{4}$$

$$C = \left(\frac{\sqrt{5} + \sqrt{2}}{2}; \frac{7 + 2\sqrt{10}}{4}\right)$$

$$P_{ABC} = \frac{1}{2}\left\|\begin{matrix} 1 & 1 & 1 \\ \sqrt{2} & \sqrt{5} & \frac{\sqrt{5} + \sqrt{2}}{2} \\ 2 & 5 & \frac{7 + 2\sqrt{10}}{4} \end{matrix}\right\| =$$

$$=\frac{1}{2}\left|\frac{7\sqrt{5}+10\sqrt{2}}{4}+\sqrt{5}+\sqrt{2}+5\sqrt{2}-2\sqrt{5}-\frac{5\sqrt{5}+5\sqrt{2}}{2}-\frac{7\sqrt{2}+4\sqrt{5}}{4}\right|=$$

$$=\frac{1}{2}\left|\frac{-7\sqrt{5}-7\sqrt{2}}{4}-\sqrt{5}+6\sqrt{2}\right|=\frac{7\sqrt{5}+7\sqrt{2}}{8}+\frac{\sqrt{5}}{2}-3\sqrt{2}$$

$$P=\frac{4}{3}\cdot P_{ABC}=\frac{4}{3}\cdot\left(\frac{7\sqrt{5}+7\sqrt{2}}{8}+\frac{\sqrt{5}}{2}-3\sqrt{2}\right)=\frac{7\sqrt{5}+7\sqrt{2}}{6}+\frac{2\sqrt{5}}{3}-4\sqrt{2}$$

Zadanie 2.

Oblicz pole odcinka paraboli $y=(x-3)^2-2$ o cięciwie AB, jeśli:

a) $A=(0;7); B=(2;\,-1)$
b) $A=(1;2); B=(3;\,-2)$
c) $A=(\pi;\,\pi^2-6\pi+2); B=(4;\,-1)$
d) $A=(\sqrt{2};9-6\sqrt{2}); B=(\sqrt{5};12-6\sqrt{5})$

Rozwiązanie

a)

$$a=\frac{-1-7}{2}=-4$$

$$y=(x-3)^2-2=x^2-6x+9-2=x^2-6x+7$$

$$y'=2x-6$$

$$2x-6=-4$$

$$2x=2$$

$$x=1$$

$$y=x^2-6x+7=1-6+7=2$$

$$C=(1;2)$$

$$P_{ABC}=\frac{1}{2}\left|\begin{vmatrix}1 & 1 & 1\\ 0 & 2 & 1\\ 7 & -1 & 2\end{vmatrix}\right|=\frac{1}{2}\big||4+7-14+1|\big|=\frac{1}{2}\cdot 2=1$$

$$P=\frac{4}{3}\cdot P_{ABC}=\frac{4}{3}\cdot 1=\frac{4}{3}$$

b)

$$a=\frac{-2-2}{3-1}=\frac{-4}{2}=-2$$

$$y=(x-3)^2-2=x^2-6x+9-2=x^2-6x+7$$

$$y'=2x-6$$

$$2x-6=-2$$

$$2x=4$$

$$x=2$$

$$y=x^2-6x+7=2^2-6\cdot 2+7=-1$$

$$C=(2;-1)$$

$$P_{ABC}=\frac{1}{2}\left\|\begin{vmatrix}1 & 1 & 1\\ 1 & 3 & 2\\ 2 & -2 & -1\end{vmatrix}\right\|=\frac{1}{2}|-3+4-2-6+4+1|=\frac{1}{2}\cdot 2=1$$

$$P=\frac{4}{3}\cdot P_{ABC}=\frac{4}{3}\cdot 1=\frac{4}{3}$$

c) brak

d)

$$a=\frac{12-6\sqrt{5}-9+6\sqrt{2}}{\sqrt{5}-\sqrt{2}}=\frac{\left(3+6(\sqrt{2}-\sqrt{5})\right)(\sqrt{5}+\sqrt{2})}{(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})}=$$

$$=\frac{\left(3+6(\sqrt{2}-\sqrt{5})\right)(\sqrt{5}+\sqrt{2})}{5-2}=\frac{\left(3+6(\sqrt{2}-\sqrt{5})\right)(\sqrt{5}+\sqrt{2})}{3}=$$

$$=\left(1+2(\sqrt{2}-\sqrt{5})\right)(\sqrt{5}+\sqrt{2})=\sqrt{5}+\sqrt{2}-6$$

$$y=(x-3)^2-2=x^2-6x+9-2=x^2-6x+7$$

$$y'=2x-6$$

$$2x-6=\sqrt{5}+\sqrt{2}-6$$

$$2x=\sqrt{5}+\sqrt{2}$$

$$x=\frac{\sqrt{5}+\sqrt{2}}{2}$$

$$y=x^2-6x+7=\left(\frac{\sqrt{5}+\sqrt{2}}{2}\right)^2-6\cdot\frac{\sqrt{5}+\sqrt{2}}{2}+7=\frac{5+2\sqrt{10}+2}{4}-3\sqrt{5}-3\sqrt{2}+7$$

$$=\frac{7+2\sqrt{10}}{4}-3\sqrt{5}-3\sqrt{2}+7=\frac{35+2\sqrt{10}}{4}-3\sqrt{5}-3\sqrt{2}$$

$$C=\left(\frac{\sqrt{5}+\sqrt{2}}{2};\frac{35+2\sqrt{10}}{4}-3\sqrt{5}-3\sqrt{2}\right)$$

$$P_{ABC}=\frac{1}{2}\left\|\begin{vmatrix}1 & 1 & 1\\ \sqrt{2} & \sqrt{5} & \frac{\sqrt{5}+\sqrt{2}}{2}\\ 9-6\sqrt{2} & 12-6\sqrt{5} & \frac{35+2\sqrt{10}}{4}-3\sqrt{5}-3\sqrt{2}\end{vmatrix}\right\|=$$

$$=\frac{1}{2}\left|1\cdot\sqrt{5}\cdot\left(\frac{35+2\sqrt{10}}{4}-3\sqrt{5}-3\sqrt{2}\right)+1\cdot\frac{\sqrt{5}+\sqrt{2}}{2}\cdot(9-6\sqrt{2})+1\cdot\sqrt{2}\cdot(12-6\sqrt{5})\right.$$

$$-1\cdot\sqrt{5}\cdot(9-6\sqrt{2})-\frac{\sqrt{5}+\sqrt{2}}{2}\cdot(12-6\sqrt{5})\cdot 1$$

$$\left.-\left(\frac{35+2\sqrt{10}}{4}-3\sqrt{5}-3\sqrt{2}\right)\cdot 1\cdot\sqrt{2}\right|=$$

$$= \frac{1}{2}\left|\frac{35\sqrt{5}+10\sqrt{2}}{4} - 15 - 3\sqrt{10} + \frac{9\sqrt{5}-6\sqrt{10}+9\sqrt{2}-12}{2} + 12\sqrt{2} - 6\sqrt{10} - 9\sqrt{5} + 6\sqrt{10} - \frac{12\sqrt{5}-30+12\sqrt{2}-6\sqrt{10}}{2} - \frac{35\sqrt{2}+4\sqrt{5}}{4} + 3\sqrt{10} + 6\right| =$$

$$= \frac{1}{2}\left|\frac{17\sqrt{2}}{4} - \frac{11\sqrt{5}}{4}\right| = \frac{11\sqrt{5}-17\sqrt{2}}{8}$$

$$P = \frac{4}{3}\cdot P_{ABC} = \frac{4}{3}\cdot\frac{11\sqrt{5}-17\sqrt{2}}{8} = \frac{11\sqrt{5}-17\sqrt{2}}{6}$$

Zadanie 3.

Oblicz pole odcinka paraboli $y = (3x-2)^2 - 1$ o cięciwie AB, jeśli

a) $A = (0;3); B = (2;15)$
b) $A = (1;0); B = (3;48)$
c) $A = (\pi; 9\pi^2 - 12\pi + 3); B = (4;99)$
d) $A = (\sqrt{2}; 21 - 12\sqrt{2}); B = (\sqrt{5}; 48 - 12\sqrt{5})$

Rozwiązanie

a)

$$a = \frac{15-3}{2} = 6$$

$$y = (3x-2)^2 - 1 = 9x^2 - 12x + 4 - 1 = 9x^2 - 12x + 3$$

$$y' = 18x - 12$$

$$18x - 12 = 6$$

$$3x - 2 = 1$$

$$3x = 3$$

$$x = 1$$

$$y = 9x^2 - 12x + 3 = 9 - 12 + 3 = 0$$

$$C = (1;0)$$

$$P_{ABC} = \frac{1}{2}\left|\begin{vmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 3 & 15 & 0 \end{vmatrix}\right| = \frac{1}{2}|3 - 6 - 15| = 9$$

$$P = \frac{4}{3}\cdot 9 = 12$$

b)

$$a = \frac{48}{2} = 24$$

$$y = (3x-2)^2 - 1 = 9x^2 - 12x + 4 - 1 = 9x^2 - 12x + 3$$

$$y' = 18x - 12$$

$$18x - 12 = 24$$

$$3x - 2 = 4$$

$$3x = 6$$

$$x = 2$$

$$y = 9x^2 - 12x + 3 = 9 \cdot 2^2 - 12 \cdot 2 + 3 = 36 - 24 + 3 = 15$$

$$C = (2; 15)$$

$$P_{ABC} = \frac{1}{2}\left\| \begin{matrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 0 & 48 & 15 \end{matrix} \right\| = \frac{1}{2}|45 + 48 - 96 - 15| = \frac{1}{2}|-18| = 9$$

$$P = \frac{4}{3} \cdot 9 = 12$$

c)

$$a = \frac{99 - 9\pi^2 + 12\pi - 3}{4 - \pi} = \frac{96 - 9\pi^2 + 12\pi}{4 - \pi} = 9\pi + 24$$

$$y = (3x - 2)^2 - 1 = 9x^2 - 12x + 4 - 1 = 9x^2 - 12x + 3$$

$$y' = 18x - 12$$

$$18x - 12 = 9\pi + 24$$

$$6x - 4 = 3\pi + 8$$

$$6x = 3\pi + 12$$

$$x = \frac{1}{2}\pi + 2$$

$$y = 9x^2 - 12x + 3 = 9 \cdot \left(\frac{1}{2}\pi + 2\right)^2 - 12 \cdot \left(\frac{1}{2}\pi + 2\right) + 3 =$$

$$= 9 \cdot \left(\frac{1}{4}\pi^2 + 2\pi + 4\right) - 6\pi - 24 + 3 = \frac{9}{4}\pi^2 + 18\pi + 36 - 6\pi - 24 + 3 =$$

$$= \frac{9}{4}\pi^2 + 12\pi + 15$$

$$C = (\frac{1}{2}\pi + 2; \frac{9}{4}\pi^2 + 12\pi + 15)$$

$$P_{ABC} = \frac{1}{2}\left\| \begin{matrix} 1 & 1 & 1 \\ \pi & 4 & \frac{1}{2}\pi + 2 \\ 9\pi^2 - 12\pi + 3 & 99 & \frac{9}{4}\pi^2 + 12\pi + 15 \end{matrix} \right\| =$$

$$= \frac{1}{2}\left|1 \cdot 4 \cdot \left(\frac{9}{4}\pi^2 + 12\pi + 15\right) + 1 \cdot \left(\frac{1}{2}\pi + 2\right) \cdot (9\pi^2 - 12\pi + 3) + 1 \cdot \pi \cdot 99 - 1 \cdot 4 \right.$$
$$\left. \cdot (9\pi^2 - 12\pi + 3) - \left(\frac{1}{2}\pi + 2\right) \cdot 99 \cdot 1 - \left(\frac{9}{4}\pi^2 + 12\pi + 15\right) \cdot 1 \cdot \pi\right| =$$

$$= \frac{1}{2}\left|9\pi^2 + 48\pi + 60 + \frac{9}{2}\pi^3 - 6\pi^2 + \frac{3}{2}\pi + 18\pi^2 - 24\pi + 6 + 99\pi - 36\pi^2 + 48\pi - 12 \right.$$
$$\left. - \frac{99}{2}\pi - 198 - \frac{9}{4}\pi^3 - 12\pi^2 - 15\pi\right| =$$

$$= \frac{1}{2}\left|\frac{9}{4}\pi^3 - 17\pi^2 + 84\pi - 144\right| = \frac{9}{8}\pi^3 - \frac{17}{2}\pi^2 + 42\pi - 72$$

$$P = \frac{4}{3} \cdot \left(\frac{9}{8}\pi^3 - \frac{17}{2}\pi^2 + 42\pi - 72\right) = \frac{3}{2}\pi^3 - \frac{34}{3}\pi^2 + 56\pi - 96$$

d)

$$a=\frac{48-12\sqrt{5}-21+12\sqrt{2}}{\sqrt{5}-\sqrt{2}}=\frac{27+12(\sqrt{2}-\sqrt{5})}{\sqrt{5}-\sqrt{2}}=\frac{\left(27+12(\sqrt{2}-\sqrt{5})\right)(\sqrt{2}+\sqrt{5})}{(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})}=$$

$$=\frac{27(\sqrt{2}+\sqrt{5})-36}{3}=9(\sqrt{2}+\sqrt{5})-12$$

$$y=(3x-2)^2-1=9x^2-12x+4-1=9x^2-12x+3$$

$$y'=18x-12$$

$$18x-12=9(\sqrt{2}+\sqrt{5})-12$$

$$18x=9(\sqrt{2}+\sqrt{5})$$

$$2x=(\sqrt{2}+\sqrt{5})$$

$$x=\frac{\sqrt{2}+\sqrt{5}}{2}$$

$$y=9x^2-12x+3=9\cdot\left(\frac{\sqrt{2}+\sqrt{5}}{2}\right)^2-12\cdot\frac{\sqrt{2}+\sqrt{5}}{2}+3=$$

$$=9\cdot\frac{7+2\sqrt{10}}{4}-6\sqrt{2}-6\sqrt{5}+3=\frac{63+18\sqrt{10}}{4}-6\sqrt{2}-6\sqrt{5}+3$$

$$C=\left(\frac{\sqrt{2}+\sqrt{5}}{2};\frac{63+18\sqrt{10}}{4}-6\sqrt{2}-6\sqrt{5}+3\right)$$

$$P_{ABC}=\frac{1}{2}\left\|\begin{matrix}1 & 1 & 1\\ \sqrt{2} & \sqrt{5} & \frac{\sqrt{2}+\sqrt{5}}{2}\\ \mathbf{21-12\sqrt{2}} & \mathbf{48-12\sqrt{5}} & \frac{63+18\sqrt{10}}{4}-6\sqrt{2}-6\sqrt{5}+3\end{matrix}\right\|=$$

$$=\frac{1}{2}\left|\frac{63\sqrt{5}+90\sqrt{2}}{4}-6\sqrt{10}-30+3\sqrt{5}+\frac{21\sqrt{2}-24+21\sqrt{5}-12\sqrt{10}}{2}+48\sqrt{2}-12\sqrt{10}\right.$$
$$-21\sqrt{5}+12\sqrt{10}-24\sqrt{2}+6\sqrt{10}-24\sqrt{5}+30-\frac{63\sqrt{2}+36\sqrt{5}}{4}+12$$
$$\left.+6\sqrt{10}-3\sqrt{2}\right|=$$

$$=\frac{1}{2}\left|28\frac{1}{4}\sqrt{2}-24\frac{3}{4}\sqrt{5}-6\sqrt{10}\right|=3\sqrt{10}+12\frac{3}{8}\sqrt{5}-14\frac{1}{8}\sqrt{2}$$

$$P=\frac{4}{3}\cdot\left(3\sqrt{10}+12\frac{3}{8}\sqrt{5}-14\frac{1}{8}\sqrt{2}\right)=4\sqrt{10}+16\frac{1}{2}\sqrt{5}-18\frac{5}{6}\sqrt{2}$$

Zadanie 4.

Oblicz pole odcinka paraboli $y = (2x-3)(3x-4)$ o cięciwie AB, jeśli:

a) $A = (0; 12); B = (2; 2)$
b) $A = (1; 1); B = (3; 15)$
c) $A = (\pi; 6\pi^2 - 17\pi + 12); B = (4; 40)$
d) $A = (\sqrt{2}; 24 - 17\sqrt{2}); B = (\sqrt{5}; 42 - 17\sqrt{5})$

Rozwiązanie

a)

$$a = \frac{-10}{2} = -5$$

$$y = (2x-3)(3x-4) = 6x^2 - 8x - 9x + 12 = 6x^2 - 17x + 12$$

$$y' = 12x - 17$$

$$12x - 17 = 5$$

$$12x = 22$$

$$x = \frac{11}{6}$$

$$y = 6x^2 - 17x + 12 = 6 \cdot \left(\frac{11}{6}\right)^2 - 17 \cdot \frac{11}{6} + 12 = \frac{121}{6} - \frac{187}{6} + 12 = -11 + 12 = 1$$

$$C = \left(\frac{11}{6}; 1\right)$$

$$P_{ABC} = \frac{1}{2}\left|\begin{vmatrix} 1 & 1 & 1 \\ 0 & 2 & \frac{11}{6} \\ 12 & 2 & 1 \end{vmatrix}\right| = \frac{1}{2}\left|2 + 22 - 24 - \frac{11}{3}\right| = \frac{1}{2}\left|-\frac{11}{3}\right| = \frac{11}{6}$$

$$P = \frac{4}{3} \cdot P_{ABC} = \frac{4}{3} \cdot \frac{11}{6} = \frac{22}{9}$$

b)

$$a = \frac{14}{2} = 7$$

$$y = (2x-3)(3x-4) = 6x^2 - 8x - 9x + 12 = 6x^2 - 17x + 12$$

$$y' = 12x - 17$$

$$12x - 17 = 7$$

$$12x = 24$$

$$x = 2$$

$$y = 6x^2 - 17x + 12 = 6 \cdot 2^2 - 17 \cdot 2 + 12 = 24 - 34 + 12 = 2$$

$$C = (2; 2)$$

$$P_{ABC}=\frac{1}{2}\left\|\begin{vmatrix}1&1&1\\1&3&2\\1&15&2\end{vmatrix}\right\|=\frac{1}{2}|6+2+15-3-30-2|=\frac{1}{2}|-12|=6$$

$$P=\frac{4}{3}\cdot P_{ABC}=\frac{4}{3}\cdot 6=8$$

c)

$$a=\frac{40-6\pi^2+17\pi-12}{4-\pi}=\frac{28-6\pi^2+17\pi}{4-\pi}=6\pi+7$$

$$y=(2x-3)(3x-4)=6x^2-8x-9x+12=6x^2-17x+12$$

$$y'=12x-17$$

$$12x-17=6\pi+7$$

$$12x=6\pi+24$$

$$x=\frac{1}{2}\pi+2$$

$$y=6x^2-17x+12=6\cdot\left(\frac{1}{2}\pi+2\right)^2-17\cdot\left(\frac{1}{2}\pi+2\right)+12=$$

$$=6\cdot\left(\frac{1}{4}\pi^2+2\pi+4\right)-\frac{17}{2}\pi+34+12=\frac{3}{2}\pi^2+12\pi+24-\frac{17}{2}\pi+48=$$

$$=\frac{3}{2}\pi^2+\frac{7}{2}\pi+72$$

$$C=\left(\frac{1}{2}\pi+2;\frac{3}{2}\pi^2+\frac{7}{2}\pi+72\right)$$

$$P_{ABC}=\frac{1}{2}\left\|\begin{vmatrix}1&1&1\\\pi&4&\frac{1}{2}\pi+2\\6\pi^2-17\pi+12&40&\frac{3}{2}\pi^2+\frac{7}{2}\pi+72\end{vmatrix}\right\|=$$

$$=\frac{1}{2}\Big|6\pi^2+14\pi+288+3\pi^3-\frac{17}{2}\pi^2+6\pi+12\pi^2-34\pi+24+40\pi-24\pi^2+68\pi$$
$$-48-20\pi-80-\frac{3}{2}\pi^3-\frac{7}{2}\pi^2-72\pi\Big|=$$

$$=\frac{1}{2}\left|\frac{3}{2}\pi^3-18\pi^2+2\pi+184\right|=\frac{3}{4}\pi^3-9\pi^2+\pi+92$$

$$P=\frac{4}{3}\cdot P_{ABC}=\frac{4}{3}\cdot\left(\frac{3}{4}\pi^3-9\pi^2+\pi+92\right)=\pi^3-12\pi^2+\frac{4}{3}\pi+\frac{368}{3}$$

d)

$$a=\frac{42-17\sqrt{5}-24+17\sqrt{2}}{\sqrt{5}-\sqrt{2}}=\frac{\left(18+17(\sqrt{2}-\sqrt{5})\right)(\sqrt{2}+\sqrt{5})}{(\sqrt{5}-\sqrt{2})(\sqrt{2}+\sqrt{5})}=$$

$$=\frac{18\sqrt{2}+18\sqrt{5}-17\cdot 3}{3}=6\sqrt{2}+6\sqrt{5}-17$$

$$y = (2x-3)(3x-4) = 6x^2 - 8x - 9x + 12 = 6x^2 - 17x + 12$$

$$y' = 12x - 17$$

$$12x - 17 = 6\sqrt{2} + 6\sqrt{5} - 17$$

$$12x = 6\sqrt{2} + 6\sqrt{5}$$

$$x = \frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{5}$$

$$y = 6x^2 - 17x + 12 = 6 \cdot \left(\frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{5}\right)^2 - 17 \cdot \left(\frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{5}\right) + 12 =$$

$$= 6 \cdot \left(\frac{1}{2} + \frac{5}{4} + \frac{1}{2}\sqrt{10}\right) - \frac{17}{2}\sqrt{2} - \frac{17}{2}\sqrt{5} + 12 = 6 \cdot \left(\frac{7}{4} + \frac{1}{2}\sqrt{10}\right) - \frac{17}{2}\sqrt{2} - \frac{17}{2}\sqrt{5} + 12 =$$

$$= \frac{21}{2} + 3\sqrt{10} - \frac{17}{2}\sqrt{2} - \frac{17}{2}\sqrt{5} + 12 = 3\sqrt{10} - \frac{17}{2}\sqrt{5} - \frac{17}{2}\sqrt{2} + \frac{45}{2}$$

$$C = \left(\frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{5}; 3\sqrt{10} - \frac{17}{2}\sqrt{5} - \frac{17}{2}\sqrt{2} + \frac{45}{2}\right)$$

$$P_{ABC} = \frac{1}{2}\left|\begin{vmatrix} 1 & 1 & 1 \\ \sqrt{2} & \sqrt{5} & \frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{5} \\ 24 - 17\sqrt{2} & 42 - 17\sqrt{5} & 3\sqrt{10} - \frac{17}{2}\sqrt{5} - \frac{17}{2}\sqrt{2} + \frac{45}{2} \end{vmatrix}\right| =$$

$$= \frac{1}{2}\left|15\sqrt{2} - \frac{85}{2} - \frac{17}{2}\sqrt{10} + \frac{45}{2}\sqrt{5} + 12\sqrt{2} - 17 + 12\sqrt{5} - \frac{17}{2}\sqrt{10} + 42\sqrt{2} - 17\sqrt{10} - 24\sqrt{5} + 17\sqrt{10} - 21\sqrt{2} + \frac{17}{2}\sqrt{10} - 21\sqrt{5} + \frac{85}{2} - 6\sqrt{5} + \frac{17}{2}\sqrt{10} + 17 - \frac{45}{2}\sqrt{2}\right| =$$

$$= \frac{1}{2}\left|-\frac{33}{2}\sqrt{5} + \frac{51}{2}\sqrt{2}\right| = \frac{33}{4}\sqrt{5} - \frac{51}{4}\sqrt{2}$$

$$P = \frac{4}{3} \cdot P_{ABC} = \frac{4}{3} \cdot \left(\frac{33}{4}\sqrt{5} - \frac{51}{4}\sqrt{2}\right) = 11\sqrt{5} - 17\sqrt{2}$$