Częściowe sumy

Wyprowadź wzory na sumy następujących szeregów

a) $W(n) = \sum_{i=1}^{n} 2i$

b) $W(n) = \sum_{i=1}^{n}(2i - 1)$

c) $W(n) = \sum_{i=1}^{n}(2i)^2$

d) $W(n) = \sum_{i=1}^{n}(2i - 1)^2$

e) $W(n) = \sum_{i=1}^{n}(2i)^3$

f) $W(n) = \sum_{i=1}^{n}(2i - 1)^3$

Rozwiązania

a)

$$W(1) = 2$$

$$W(2) = 6$$

$$W(3) = 12$$

$$W(4) = 20$$

$$W(5) = 30$$

$$W(6) = 42$$

$$W(7) = 56$$

$$\begin{cases} a + b + c = 2 \\ 4a + 2b + c = 6 \\ 9a + 3b + c = 12 \end{cases}$$

$$\begin{cases} a + b + c = 2 \\ 3a + b = 4 \\ 5a + b = 6 \end{cases}$$

$$\begin{cases} a + b + c = 2 \\ 3a + b = 4 \\ 2a = 2 \end{cases}$$

$$\begin{cases} a + b + c = 2 \\ 3a + b = 4 \\ a = 1 \end{cases}$$

$$\begin{cases} a + b + c = 2 \\ 3 + b = 4 \\ a = 1 \end{cases}$$

$$\begin{cases} a + b + c = 2 \\ b = 1 \\ a = 1 \end{cases}$$

$$\begin{cases} 2 + c = 2 \\ b = 1 \\ a = 1 \end{cases}$$

$$\begin{cases} c = 0 \\ b = 1 \\ a = 1 \end{cases}$$

$$\sum_{i=1}^{n} 2i = n^2 + n = n(n + 1)$$

b)

$$W(1) = 1$$

$$W(2) = 4$$

$$W(3) = 9$$

$$W(4) = 16$$

$$W(5) = 25$$

$$W(6) = 36$$

$$W(7) = 49$$

$$\begin{cases} a + b + c = 1 \\ 4a + 2b + c = 4 \\ 9a + 3b + c = 9 \end{cases}$$

$$\begin{cases} a + b + c = 1 \\ 3a + b = 3 \\ 5a + b = 5 \end{cases}$$

$$\begin{cases} a + b + c = 1 \\ 3a + b = 3 \\ 2a = 2 \end{cases}$$

$$\begin{cases} a + b + c = 1 \\ 3a + b = 3 \\ a = 1 \end{cases}$$

$$\begin{cases} a + b + c = 1 \\ 3 + b = 3 \\ a = 1 \end{cases}$$

$$\begin{cases} a + b + c = 1 \\ b = 0 \\ a = 1 \end{cases}$$

$$\begin{cases} 1 + c = 1 \\ b = 0 \\ a = 1 \end{cases}$$

$$\begin{cases} c = 0 \\ b = 0 \\ a = 1 \end{cases}$$

$$\sum_{i=1}^{n} (2i - 1) = \boldsymbol{n^2}$$

c)

$$W(1) = 4$$

$$W(2) = 20$$

$$W(3) = 56$$

$$W(4) = 120$$

$$W(5) = 220$$

$$W(6) = 364$$

$$W(7) = 560$$

$$\begin{cases} a + b + c + d = 4 \\ 8a + 4b + 2c + d = 20 \\ 27a + 9b + 3c + d = 56 \\ 64a + 16b + 4c + d = 120 \end{cases}$$

$$\begin{cases} a + b + c + d = 4 \\ 7a + 3b + c = 16 \\ 19a + 5b + c = 36 \\ 37a + 7b + c = 64 \end{cases}$$

$$\begin{cases} a + b + c + d = 4 \\ 7a + 3b + c = 16 \\ 12a + 2b = 20 \\ 18a + 2b = 28 \end{cases}$$

$$\begin{cases} a + b + c + d = 4 \\ 7a + 3b + c = 16 \\ 12a + 2b = 20 \\ 3a = 4 \end{cases}$$

$$\begin{cases} a + b + c + d = 4 \\ 7a + 3b + c = 16 \\ 12a + 2b = 20 \\ a = 1\frac{1}{3} \end{cases}$$

$$\begin{cases} 1\frac{1}{3} + b + c + d = 4 \\ 9\frac{1}{3} + 3b + c = 16 \\ 16 + 2b = 20 \\ a = 1\frac{1}{3} \end{cases}$$

$$\begin{cases} b + c + d = 2\frac{2}{3} \\ 3b + c = 6\frac{2}{3} \\ 2b = 4 \\ a = 1\frac{1}{3} \end{cases}$$

$$\begin{cases} b + c + d = 2\dfrac{2}{3} \\ 3b + c = 6\dfrac{2}{3} \\ b = 2 \\ a = 1\dfrac{1}{3} \end{cases}$$

$$\begin{cases} 2 + c + d = 2\dfrac{2}{3} \\ 6 + c = 6\dfrac{2}{3} \\ b = 2 \\ a = 1\dfrac{1}{3} \end{cases}$$

$$\begin{cases} c + d = \dfrac{2}{3} \\ c = \dfrac{2}{3} \\ b = 2 \\ a = 1\dfrac{1}{3} \end{cases}$$

$$\begin{cases} \dfrac{2}{3} + d = \dfrac{2}{3} \\ c = \dfrac{2}{3} \\ b = 2 \\ a = 1\dfrac{1}{3} \end{cases}$$

$$\begin{cases} d = 0 \\ c = \dfrac{2}{3} \\ b = 2 \\ a = 1\dfrac{1}{3} \end{cases}$$

$$\sum_{i=1}^{n} (2i)^2 = 1\frac{1}{3}n^3 + 2n^2 + \frac{2}{3}n = 2n\left(\frac{2}{3}n^2 + n + \frac{1}{3}\right)$$

d)

$$W(1) = 1$$

$$W(2) = 10$$

$$W(3) = 35$$

$$W(4) = 84$$

$$W(5) = 165$$

$$W(6) = 286$$

$$W(7) = 455$$

$$\begin{cases} a + b + c + d = 1 \\ 8a + 4b + 2c + d = 10 \\ 27a + 9b + 3c + d = 35 \\ 64a + 16b + 4c + d = 84 \end{cases}$$

$$\begin{cases} a + b + c + d = 1 \\ 7a + 3b + c = 9 \\ 19a + 5b + c = 25 \\ 37a + 7b + c = 49 \end{cases}$$

$$\begin{cases} a + b + c + d = 1 \\ 7a + 3b + c = 9 \\ 12a + 2b = 16 \\ 18a + 2b = 24 \end{cases}$$

$$\begin{cases} a + b + c + d = 1 \\ 7a + 3b + c = 9 \\ 16a + b = 18 \\ 3a = 4 \end{cases}$$

$$\begin{cases} a + b + c + d = 1 \\ 7a + 3b + c = 9 \\ 16a + b = 18 \\ a = 1\frac{1}{3} \end{cases}$$

$$\begin{cases} 1\frac{1}{3} + b + c + d = 1 \\ 9\frac{1}{3} + 3b + c = 9 \\ 21\frac{1}{3} + b = 18 \\ a = 1\frac{1}{3} \end{cases}$$

$$\begin{cases} b + c + d = -\frac{1}{3} \\ 3b + c = -\frac{1}{3} \\ b = -3\frac{1}{3} \\ a = 1\frac{1}{3} \end{cases}$$

$$\begin{cases} -3\frac{1}{3} + c + d = -\frac{1}{3} \\ -10 + c = -\frac{1}{3} \\ b = -3\frac{1}{3} \\ a = 1\frac{1}{3} \end{cases}$$

$$\begin{cases} c + d = 3 \\ c = 9\frac{2}{3} \\ b = -3\frac{1}{3} \\ a = 1\frac{1}{3} \end{cases}$$

$$\begin{cases} 9\frac{2}{3} + d = 3 \\ c = 9\frac{2}{3} \\ b = -3\frac{1}{3} \\ a = 1\frac{1}{3} \end{cases}$$

$$\begin{cases} d = -6\frac{2}{3} \\ c = 9\frac{2}{3} \\ b = -3\frac{1}{3} \\ a = 1\frac{1}{3} \end{cases}$$

$$\sum_{i=1}^{n}(2i-1)^2 = 1\frac{1}{3}n^3 - 3\frac{1}{3}n^2 + 9\frac{2}{3}n - 6\frac{2}{3}$$

e)

$$W(1) = 8$$

$$W(2) = 72$$

$$W(3) = 288$$

$$W(4) = 800$$

$$W(5) = 1800$$

$$W(6) = 3528$$

$$W(7) = 6272$$

$$\begin{cases} a + b + c + d + e = 8 \\ 16a + 8b + 4c + 2d + e = 72 \\ 81a + 27b + 9c + 3d + e = 288 \\ 256a + 64b + 16c + 4d + e = 800 \\ 625a + 125b + 25c + 5d + e = 1800 \end{cases}$$

$$\begin{cases} a + b + c + d + e = 8 \\ 15a + 7b + 3c + d = 64 \\ 65a + 19b + 5c + d = 216 \\ 175a + 37b + 7c + d = 512 \\ 369a + 61b + 9c + d = 1000 \end{cases}$$

$$\begin{cases} a + b + c + d + e = 8 \\ 15a + 7b + 3c + d = 64 \\ 50a + 12b + 2c = 152 \\ 110a + 18b + 2c = 296 \\ 194a + 24b + 2c = 488 \end{cases}$$

$$\begin{cases} a + b + c + d + e = 8 \\ 15a + 7b + 3c + d = 64 \\ 25a + 6b + c = 76 \\ 55a + 9b + c = 148 \\ 97a + 12b + c = 244 \end{cases}$$

$$\begin{cases} a + b + c + d + e = 8 \\ 15a + 7b + 3c + d = 64 \\ 25a + 6b + c = 76 \\ 30a + 3b = 72 \\ 42a + 3b = 96 \end{cases}$$

$$\begin{cases} a + b + c + d + e = 8 \\ 15a + 7b + 3c + d = 64 \\ 25a + 6b + c = 76 \\ 10a + b = 24 \\ 14a + b = 32 \end{cases}$$

$$\begin{cases} a + b + c + d + e = 8 \\ 15a + 7b + 3c + d = 64 \\ 25a + 6b + c = 76 \\ 10a + b = 24 \\ 4a = 8 \end{cases}$$

$$\begin{cases} a + b + c + d + e = 8 \\ 15a + 7b + 3c + d = 64 \\ 25a + 6b + c = 76 \\ 10a + b = 24 \\ a = 2 \end{cases}$$

$$\begin{cases} 2 + b + c + d + e = 8 \\ 30 + 7b + 3c + d = 64 \\ 50 + 6b + c = 76 \\ 20 + b = 24 \\ a = 2 \end{cases}$$

$$\begin{cases} b + c + d + e = 6 \\ 7b + 3c + d = 34 \\ 6b + c = 26 \\ b = 4 \\ a = 2 \end{cases}$$

$$\begin{cases} c + d + e = 2 \\ 3c + d = 6 \\ c = 2 \\ b = 4 \\ a = 2 \end{cases}$$

$$\begin{cases} 2 + d + e = 2 \\ 6 + d = 6 \\ c = 2 \\ b = 4 \\ a = 2 \end{cases}$$

$$\begin{cases} d + e = 0 \\ d = 0 \\ c = 2 \\ b = 4 \\ a = 2 \end{cases}$$

$$\begin{cases} e = 0 \\ d = 0 \\ c = 2 \\ b = 4 \\ a = 2 \end{cases}$$

$$\sum_{i=1}^{n} (2i)^3 = 2n^4 + 4n^3 + 2n^2 = 2n^2(n^2 + 2n + 1) = 2n^2(n + 1)^2$$

f)

$$W(1) = 1$$

$$W(2) = 28$$

$$W(3) = 153$$

$$W(4) = 496$$

$$W(5) = 1225$$

$$W(6) = 2556$$

$$W(7) = 4753$$

$$\begin{cases} a + b + c + d + e = 1 \\ 16a + 8b + 4c + 2d + e = 28 \\ 81a + 27b + 9c + 3d + e = 153 \\ 256a + 64b + 16c + 4d + e = 496 \\ 625a + 125b + 25c + 5d + e = 1225 \end{cases}$$

$$\begin{cases} a + b + c + d + e = 1 \\ 15a + 7b + 3c + d = 27 \\ 65a + 19b + 5c + d = 125 \\ 175a + 37b + 7c + d = 343 \\ 369a + 61b + 9c + d = 729 \end{cases}$$

$$\begin{cases} a + b + c + d + e = 1 \\ 15a + 7b + 3c + d = 27 \\ 50a + 12b + 2c = 98 \\ 110a + 18b + 2c = 218 \\ 194a + 24b + 2c = 386 \end{cases}$$

$$\begin{cases} a + b + c + d + e = 1 \\ 15a + 7b + 3c + d = 27 \\ 25a + 6b + c = 49 \\ 55a + 9b + c = 109 \\ 97a + 12b + c = 193 \end{cases}$$

$$\begin{cases} a + b + c + d + e = 1 \\ 15a + 7b + 3c + d = 27 \\ 25a + 6b + c = 49 \\ 30a + 3b = 60 \\ 42a + 3b = 84 \end{cases}$$

$$\begin{cases} a + b + c + d + e = 1 \\ 15a + 7b + 3c + d = 27 \\ 25a + 6b + c = 49 \\ 10a + b = 20 \\ 14a + 3b = 28 \end{cases}$$

$$\begin{cases} a + b + c + d + e = 1 \\ 15a + 7b + 3c + d = 27 \\ 25a + 6b + c = 49 \\ 16a = 32 \\ 14a + 3b = 28 \end{cases}$$

$$\begin{cases} a + b + c + d + e = 1 \\ 15a + 7b + 3c + d = 27 \\ 25a + 6b + c = 49 \\ a = 2 \\ 14a + 3b = 28 \end{cases}$$

$$\begin{cases} 2 + b + c + d + e = 1 \\ 30 + 7b + 3c + d = 27 \\ 50 + 6b + c = 49 \\ a = 2 \\ 28 + 3b = 28 \end{cases}$$

$$\begin{cases} b + c + d + e = -1 \\ 7b + 3c + d = -3 \\ 6b + c = -1 \\ a = 2 \\ 3b = 0 \end{cases}$$

$$\begin{cases} b + c + d + e = -1 \\ 7b + 3c + d = -3 \\ 6b + c = -1 \\ a = 2 \\ b = 0 \end{cases}$$

$$\begin{cases} c + d + e = -1 \\ 3c + d = -3 \\ c = -1 \\ a = 2 \\ b = 0 \end{cases}$$

$$\begin{cases} -1 + d + e = -1 \\ -3 + d = -3 \\ c = -1 \\ a = 2 \\ b = 0 \end{cases}$$

$$\begin{cases} d + e = 0 \\ d = 0 \\ c = -1 \\ a = 2 \\ b = 0 \end{cases}$$

$$\begin{cases} e = 0 \\ d = 0 \\ c = -1 \\ a = 2 \\ b = 0 \end{cases}$$

$$\sum_{i=1}^{n} (2i - 1)^3 = 2n^4 - n^2 = n^2(2n^2 - 1)$$